

# Case Study: Teaching High School Mathematics by Origami

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## Abstract

In this paper a certain optimise problem which used to be discussed by the tool of calculus in college is re-discussed in a new way, i.e. with the help of origami and wire-frame soap test.

It has been noticed many years ago by scientists that a honeycomb-cell has certain optimise property. It has the least surface area among those hexagonal prism of unit edge-length and fixed volume. However the proof remain difficult to understand since the mathematical model is somewhat tricky to establish. Even if the equation of the problem has been given, one can not easily find out its extremum without the tool of calculus. It is a great pity to exclude such an interesting topic out of high school class.

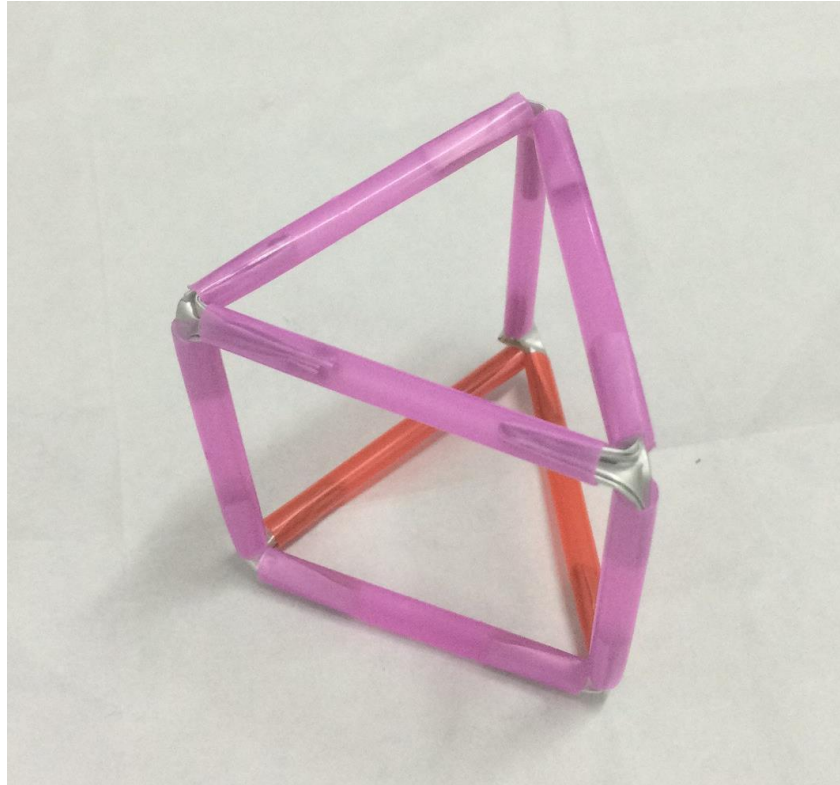
Actually there exists another difficulty in attracting the high school students to this problem. Since students may not have been observed a real honeycomb, they may have no ideas on what it is like.

So a high school mathematics teacher can introduce the honeycomb cell by the help of origami. Such a model can be easily made from a sheet of A4 paper in less than 20 steps. That means students will get a model of their own in half an hour and costs no money. Fig. 1.



**Figure 1:** A honeycomb cell folded from a sheet of A4 paper

Only when students have the model in hand can they begin thinking why such a roof yields a minimum surface. The result is exciting when students finally get a proof without words. The students will do a foam-film test to verify what the teacher claims before class. The test will need a simple hand-made device, a straw frame of a triangular prism. Fig. 2. It is as easy.



**Figure 2:** straw made frame to be used in soap film test

Based on the test evidence, students can easily confirm that a honeycomb cell really has a minimum surface and they would like to use mathematics tool to give a rigid proof which is relatively easy.

Since a soap film has top and bottom collapsed inward and has its interlines forming exactly the equal four angles in space, it is obviously that this angle (equals to  $109.47^\circ$  approximately, and precisely it is  $\cos^{-1}(-1/3)$ ) coincide with that of on the roof of a honeycomb cell.

The mathematics proof can also have a simple version like this:

To find a minimum of the function  $y = 3\sqrt{3}a\sqrt{x^2 + \frac{a^2}{4}} - 3ax - \frac{3\sqrt{3}}{2}a^2$ , it suffices using  $\Delta \geq 0$  to simplified equation  $18a^2x^2 - (6ay + 9\sqrt{3}a^3)x - (3\sqrt{3}a^2y + y^2) = 0$ ,

to reach  $y_{min} = \frac{3}{2}(\sqrt{2} - \sqrt{3})a^2$ . That implied roof angle equals to  $109.47^\circ$ .